

Understanding Multiplication Across the Grades

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Mathematics Education Collaborative (MEC)
Under the Leadership of Ruth E. Parker

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Getting Started



Purpose of this Session

Understanding Multiplication Across the Grades is designed to help parents learn about how multiplication can be taught for understanding. Parents experience ways to teach multiplication in important and engaging mathematical contexts such as probability, geometry, and data analysis. Fun and engaging ways to provide practice with basic facts are highlighted. The difference between a focus on rote memorization and a focus on knowing and understanding is highlighted. The content of this session is appropriate for teachers and parents of grades 3 through 8, and students of grades 4–8.

Introduction

An informed public is essential if necessary changes in the teaching of mathematics are to occur on a broad scale. Professional organizations such as the National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), and the Mathematical Sciences Education Board (MSEB) have called for broad and sweeping changes in the mathematics content taught at the K–12 levels, in the learning environment, in the role of the teacher in the mathematics classroom, and in methods of assessing mathematical understandings. Sustained efforts to support teachers and schools in making these changes have been launched on many fronts over the past decade. Sound and comprehensive mathematics programs have been developed. Yet most mathematics reform efforts to date have fallen far short of the vision put forth by the mathematics education community.

One reason increasingly recognized as a contributing factor to this lack of success has been our failure to adequately engage with parents and the public in ways that help them understand important issues in mathematics education, the need for change, and the nature of changes needed. Misinformation about mathematics education is prevalent in today's media. Strategic attempts to organize parents in opposition to reform-based programs

are prevalent and are negatively impacting mathematics restructuring efforts taking place in many locations throughout the country. Although the need for working with parents and the public is widely recognized, many teachers and administrators feel inadequately prepared to work with parents in support of quality mathematics programs in their schools.

In response to this need, the Mathematics Education Collaborative (MEC) was founded in 1999 to strategically aid schools and school districts in working with their parents and public in support of quality mathematics programs. Ruth Parker, CEO of MEC, has worked with parents and the public in more than two hundred communities throughout the nation during the past eight years. A National Science Foundation grant (ESI#9908602) has supported MEC's work with parents and the public in three regions of the country during the past five years. The grant also supported the development of *Supporting School Mathematics: How to Work with Parents and the Public*, a series of six interactive sessions designed to help parents and the public understand some fundamental changes that have taken place, or need to take place, in mathematics classrooms. All the sessions have been extensively field-tested by teacher leaders in school districts throughout the nation. The six sessions, which are available separately or as a package, include the following:

- ◆ *Understanding Addition and Subtraction in the Primary Grades*
- ◆ *Encouraging Mathematical Thinkers: The Basics and More*
- ◆ *Understanding Multiplication Across the Grades*
- ◆ *Understanding Fractions Across the Grades*
- ◆ *Helping with Math at Home: Ideas for Parents*
- ◆ *Helping with Math at Home: More Ideas for Parents*

Included with each session is *A Planning Handbook for Presenters*, developed to assist you as you plan and prepare for presenting any and all of the six sessions.

Using the Parent Sessions

These parent sessions are designed to complement the multifaceted work that a school district does to improve mathematics education. They are

designed specifically to help parents and the public understand important issues that surround mathematics education today. This series of sessions will help parents learn to recognize quality mathematics programs, support the work of teachers and administrators, and advocate for strong mathematics programs in their schools. These sessions are designed to help teachers and administrators as they prepare to work with parents and the public. The sessions are intended to give parents opportunities to learn about strong mathematics programs. They are set up a little differently than some of the more familiar school math events. It is critical to the sessions that the presenters go beyond the activities and give the audience a chance to think about the issues involved. Each session is based on a few important ideas that are revisited throughout the session. Presenters need to familiarize themselves with these big ideas and use the sessions to help parents understand them. Presenting the sessions as a series of activities without the important discussions, while fun and interesting, will not accomplish the intended goals. It is important to be clear about the main messages in each session and about the mathematical goals.

These sessions are designed to bring about new insights for parents. They have been designed to run between ninety minutes and two hours. They have worked successfully with groups as small as fifteen and as large as one hundred or more. Although participants are actively engaged as learners, the sessions generally require a minimal amount of manipulative materials and technology.

Planning the Sessions

A Planning Handbook for Presenters has been developed to assist you as you plan and prepare for presenting any or all of the six parent sessions. It includes practical advice you will want to consider on topics such as getting parents to come, child care, addressing language needs, and engaging parents during the session. A major part of the handbook addresses questions that parents commonly ask about math in classrooms, along with sample responses. One section of the handbook addresses the issue of invented and alternative algorithms. This section is included because the issue is so central to changes occurring and to the concerns some parents have about elementary mathematics education. You will find that this handbook is also useful when you are developing your own sessions.

Making the Sessions Your Own

It is important for you as a presenter to know the session well and, in as many ways as possible, make it your own. This may mean changing some of the anecdotes and telling your own stories. It may mean using your own student work. It may mean talking about your own experiences in the classroom with students. Using personal examples to illustrate points helps your presentation “ring true” and be more convincing to parents.

Addressing Local Needs

You will need to customize the presentation to address local issues so the audience can connect to the message. For example, find out what textbooks and what assessments are used locally. Try to understand whether and in what direction the district or school is trying to move. Try to anticipate the questions and concerns of the parents. More information about these issues can be found in *A Planning Handbook for Presenters*.

Student Work

Many of the sessions call for the use of student work. Student work is another powerful tool to use in convincing parents about the importance of a strong mathematics program. Yet it is also difficult for a presenter to be knowledgeable about student work when it comes from someone else’s classroom. In some sessions, one example of student work is included to show what the assignment might look like, but it is assumed you will collect your own student work to discuss during the session.

Because it is important that you become familiar with the games and/or activities before presenting them to parents, you will want to try them with a group of students first. This will allow you to gather student work and anecdotes for your presentation. It will also familiarize you with questions that come up during the activities and help you troubleshoot the directions. You will then be able to more confidently answer parents’ questions about the assignment and the work.

Understanding the Mathematics

Some of the sessions also include a section about the mathematics of the problem presented. The mathematical explanations are included when a

problem is apt to be misunderstood. If you are presenting one of these sessions, you will need to explore the mathematics and become comfortable with the theoretical ideas behind the problem. Though you may not need to provide a mathematical explanation during the session, you will want to be prepared to do so in case you are asked by parents about the mathematics of the problem.

Adapting the Sessions to Use with Teachers

The content of these sessions is also very appropriate for teachers and administrators who are learning more about mathematics education. The sessions can be easily adapted to use for professional development by reviewing the session ahead of time and changing some of the language that is used; for instance, use *participant* or *teacher* rather than *parent*. In addition, the focus of the session and the issues addressed may need to be slightly modified for teachers. You might want to choose more appropriate quotes that have more meaning for the classroom. It is a good idea to go through the entire session considering your specific audience and make adjustments where necessary.

Research

Many times parents have questions about the research that exists on a particular mathematical issue. There is a user-friendly and useful research-based overview of the issues in the document *Teaching and Learning Mathematics: Using Research to Shift from the “Yesterday” Mind to the “Tomorrow” Mind*. It was written by Dr. Jerry Johnson and can be found at the following website: www.k12.wa.us. A free copy of the document can be obtained by placing an order on the website or by calling 888-595-3276. You can choose to use information from this source for your presentation or give parents access to the document itself. Another resource for current articles about mathematics education is www.mathematicallysane.com. You may also want to start your own collection of articles that you think will be useful for parents to read and discuss.

Other Resources

Included in this introduction are two additional resources that might be helpful to presenters of this session. The first is the definition of “mathematical

proficiency” from the book *Adding It Up: Helping Children Learn Mathematics* (Figure 1, p. 6) written by the National Research Council. Mathematical proficiency is visualized as five strands of mathematics—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition—that are interdependent and interwoven to form a sturdy, tightly connected representation of mathematical proficiency. As a visual, it may be useful in helping parents understand *all* the different aspects involved in creating mathematically powerful students.

The second is the chart “What Can Parents Do?” developed by MEC’s Public Session Development Team (p. 7). It outlines the roles parents can assume in their child’s mathematical education. Though the chart had originally been intended to use with parents, it seemed cumbersome; however, it may help a presenter think about the goals and focus of a particular session, or it may be adapted to use with parents in shorter sessions over a longer period of time or series of workshops. With this in mind, it also is included in the handout section of each session.

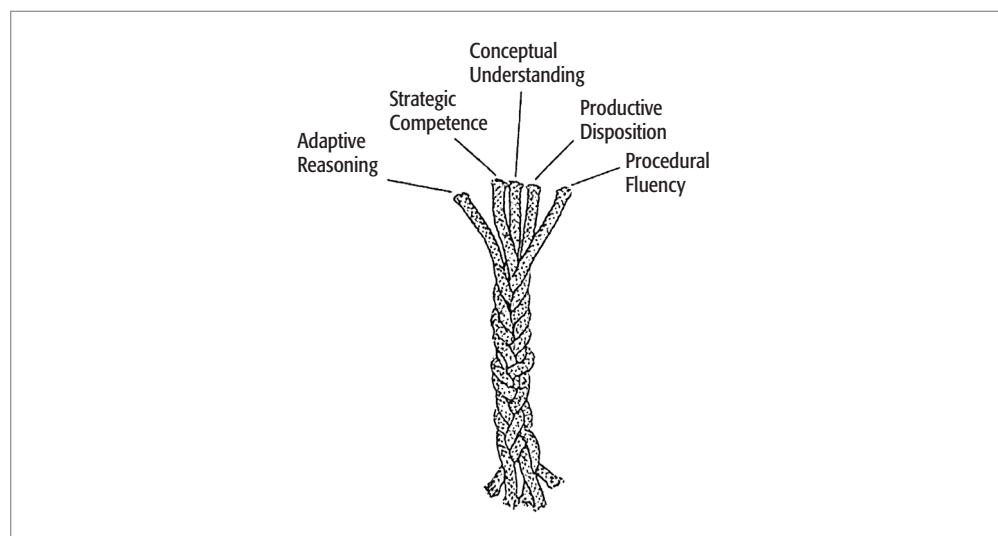


FIGURE 1: *Intertwined Strands of Proficiency*

What Can Parents Do . . .

	. . . as an Adult?	. . . as a Parent?	. . . with Your Child at Home?	. . . for Your Child at School?
Parent as a Learner	<ul style="list-style-type: none"> • continue to learn and be a learner of mathematics • recognize that mathematics is an important tool for making sense of the world around you • recognize that new discoveries are still being made in mathematics 	<ul style="list-style-type: none"> • learn what math is all about for your child • pay attention to experiences that impact your child's attitudes about mathematics 	<ul style="list-style-type: none"> • investigate and play with numbers • involve your child in the measuring and comparing that you do at home • use games to support mathematical thinking • do mental arithmetic and share strategies 	<ul style="list-style-type: none"> • attend parent math nights • look for information that comes home about your school's mathematics program • become familiar with mathematics as it is taught in your child's classroom
Parent as a Researcher	<ul style="list-style-type: none"> • be curious about how you and others solve problems • broaden your understanding of what mathematics is • find out about current research in mathematics teaching and learning 	<ul style="list-style-type: none"> • ask questions to understand your child's thinking • be curious about how your child solves problems 	<ul style="list-style-type: none"> • investigate with your child his or her mathematical questions • explore with your child the best time and place to do homework 	<ul style="list-style-type: none"> • ask questions about your child's mathematics program, both the goals and how it is taught • find out about roles for parents in the mathematics program at your child's school
Parent as a Communicator	<ul style="list-style-type: none"> • recognize that learning and understanding mathematics depends on communication (listening, talking, and writing) • learn to examine and explain your own thinking 	<ul style="list-style-type: none"> • share information with your child about how you use mathematics • display a positive disposition about mathematics 	<ul style="list-style-type: none"> • talk with your child about the importance of mathematics in his or her life • explore the many ways to approach solving problems • be clear about your expectations for homework • find ways to show enthusiasm about mathematics 	<ul style="list-style-type: none"> • find a way to let the school know what you want for your child's mathematical education • become a knowledgeable advocate for good mathematics education

Presenter's Guide



Sample Workshop Agenda



- 7:00 Welcome and Introduction
- 7:10 Multiplication Facts and Concepts
- 7:50 Working Toward Fluency
- 8:40 What About Drill and Practice?
- 8:50 Closing



Sample Workshop Agenda with Activity Detail

7:00 Introduction to Workshop

Welcome

Overview of Goals for Workshop

7:10 Multiplication Facts and Concepts

Circles and Stars

Multiplication Rectangles

Building a Multiplication Table

Designs Found in Multiples

7:50 Working Toward Fluency

Patterns in the Multiplication Table

8:40 What About Drill and Practice?

Multiplication Tic-Tac-Toe

Brian Bangel

8:50 Closing

Collect Response Forms

Thanks and Good Night

Q & A

Materials Needed



For Presenter

overhead projector

prepared overhead transparencies

blank overhead transparencies

overhead transparency pens

cutouts of transparent paper rectangles sized (1×1) , (1×2) , (1×3) , (1×4) , (2×2) , (1×5) , (1×6) , (2×3) , (1×7) , (1×8) , (2×4) , (1×10) , (2×5) , (1×9) , (3×3) , (1×12) , (2×6) , (3×4) , (1×16) , (2×8) , (4×4) to match grid in Overhead 4

underlying blank grid in Overhead 4

one die

four color tiles for overhead

ten copies of Overhead 5 colored with multiple patterns for 2, 3, 4, 5, 6, 7, 8, 9, 10, and 12.

Overhead 7 with multiples of 3 circled

two paperclips

optional: multiplication wall of student work (see p. 52)

optional: colored game chips

wet paper towels

For Participants

duplicates of handouts—one copy per person

optional: two paperclips per group of four participants

optional: colored game chips

Room arrangement should enable participants to work in pairs.



Overview and Goals

Notes for Presenter

This session has been developed for a group of parents who may or may not have attended another parent session. That is, relative to both its goals and mathematics content, this workshop is designed to stand alone.

The workshop directions are meant to guide your planning and facilitate your preparation, not dictate your words or actions. You need to make decisions about whether the structure and content make sense for your situation. The constraints of the room, time, numbers of parents, and personal style may necessitate some adjustments to various details.

Goals for the Session

Participants will:

- ◆ learn what it means to teach multiplication for understanding
- ◆ experience drill and practice in an engaging problem-solving context
- ◆ understand the distinction between knowing multiplication facts and simply memorizing them

Other ideas we hope parents encounter during this particular session:

- ◆ Mathematics is engaging and motivating.
- ◆ When children learn multiplication with deep understanding, they see how it relates to many other mathematical ideas.

So with the multiplication facts, either we can give children timed tests and flash cards and isolated drill or we can use the multiplication table to investigate patterns in multiples and explore our number system. Which approach do you think is more likely to result in students who understand multiplication, who want to do mathematics, and who enjoy it?

What About Drill and Practice? (10 MINUTES)


SAY *We're often asked the question, "What about drill and practice? Is it important?" Yes, it is important. Whatever it is that we're trying to learn to do, doing lots of it often helps us to be able to do it with ease. But with math, it is very important that we understand what drill will and will not do.*

Drill will not teach a mathematical concept. Now that is a bit hard to understand, but again, all the drill in the world will not teach a mathematical idea. Many of us taught for years thinking that it would. We often taught believing that if we just gave children one more page of problems they would get it.

We now understand that this is not where "getting it" comes from. But once we have given children opportunities to understand multiplication and how it is used, then drill and practice can help them learn to use multiplication facts or relationships with ease.


Drill and practice with mathematics should be engaging and motivating and provided in contexts that ask children to think and reason and behave in mathematically important ways. It should result in kids who want to do mathematics.

Purpose: Participants will engage in a game that is motivating and provides practice with the multiplication facts.

 One half of the room becomes the Xs (or red chips) and the other half of the room become the Os (or the blue chips).

OVERHEAD 7

Tic-Tac-Toe Products									
1	2	3	4	5	6				
7	8	9	10	12	14				
15	16	18	20	21	24				
25	27	28	30	32	35				
36	40	42	45	48	49				
54	56	63	64	72	81				
0	1	2	3	4	5	6	7	8	9

 Play the game with the whole group until each group has had at least four turns so that players get to think about strategies.

Multiplication Tic-Tac-Toe (8 out of 10 minutes)

SAY *One of my favorite games that provides drill with multiplication is in your packet. You won't need it right now, but we will play it for a few minutes so that you will know how to play it at home.*

Display Overhead 7.

Divide the audience into two teams (Xs and Os) and play Multiplication Tic-Tac-Toe, recording at the overhead.

SAY *Each team is trying to capture four numbers in a row horizontally, vertically, or diagonally. The way you capture a number is to tell me what factors you want the paper clips placed on. For example, if you said, "4 and 6," I would ask you what 4 times 6 is. If you told me 24, I would put your X or red chip on 24. After the opening move, each team can move only one paper clip. You can move it anywhere you would like, but you can only move one paper clip. It is OK to have both paper clips on the same number, such as $6 \times 6 = 36$. Let's play for a few minutes and the directions will be clear.*

Xs (or Reds), you won the toss. Where do you want to start? I need two factors.

Place the paper clips on the two factors suggested and ask for the product. Remind the Os (or Blue Team) that they can now move only one paper clip. Continue the game until people have had a chance to think about strategies (about five minutes), then cover up the overhead.

SAY *Are you doing drill? Yes, you're considering several multiplication facts for each move you make. But for obvious reasons, this is not a good way for students to play the game.*

Some of you are trying to follow a strategy and I'm taking whoever is loudest, so you might be frustrated. Also, I know that some of you are doing drill and practice. But I don't know that all of you are.

It's not OK for one-sixth of the class to be tuned out. So, in the classroom we would have children seated in groups of four, give each group a game board, and have two children play two children.

The reason for having children work in partners is two-fold. First, some children don't know their multiplication facts and it helps to talk them over as part of learning them. Second, some children don't see the strategies.

I hope we played the game long enough for you to see that this game is very sophisticated with regard to strategies—especially when you realize you not only need to pay attention to where you want to be but also to where you don't want to leave your opponents and where you might want to be positioned for your next move.

Once children catch on, the game takes a long time to play. It sometimes takes 45 minutes or more, and the wonderful thing is that children are practicing their multiplication facts for 45 minutes and no one is complaining.



If time allows, have the participants play the game for a short time.

We also send games like this home for homework, but we send it with a letter that says something such as

Dear Parents,

We are working on learning multiplication facts this year. We will be doing a lot of practice.

Please notice the practice you are doing while playing this game, and enjoy playing!

We've learned that the letters are important because even though games like this go home for homework, parents often ask why they are not seeing drill coming home. We now realize that if parents are used to seeing drill look like 40 problems on a page, they won't necessarily notice that children might be doing 400-plus such problems in the context of playing a game that also asks them to think, reason, and behave in mathematically important ways.

We have to get a lot better at understanding that drill is very solidly embedded in the new programs we have available to us today. It just looks very different from what we were used to seeing.

If you think for a moment, though, about which approach is more likely to result in a child who learns to like mathematics, the reasonable choice seems pretty obvious.